Integrated Technical Education Cluster
At AlAmeeria

J-601-1448
Electronic Principals

Lecture #7
BJT and JFET Frequency Response

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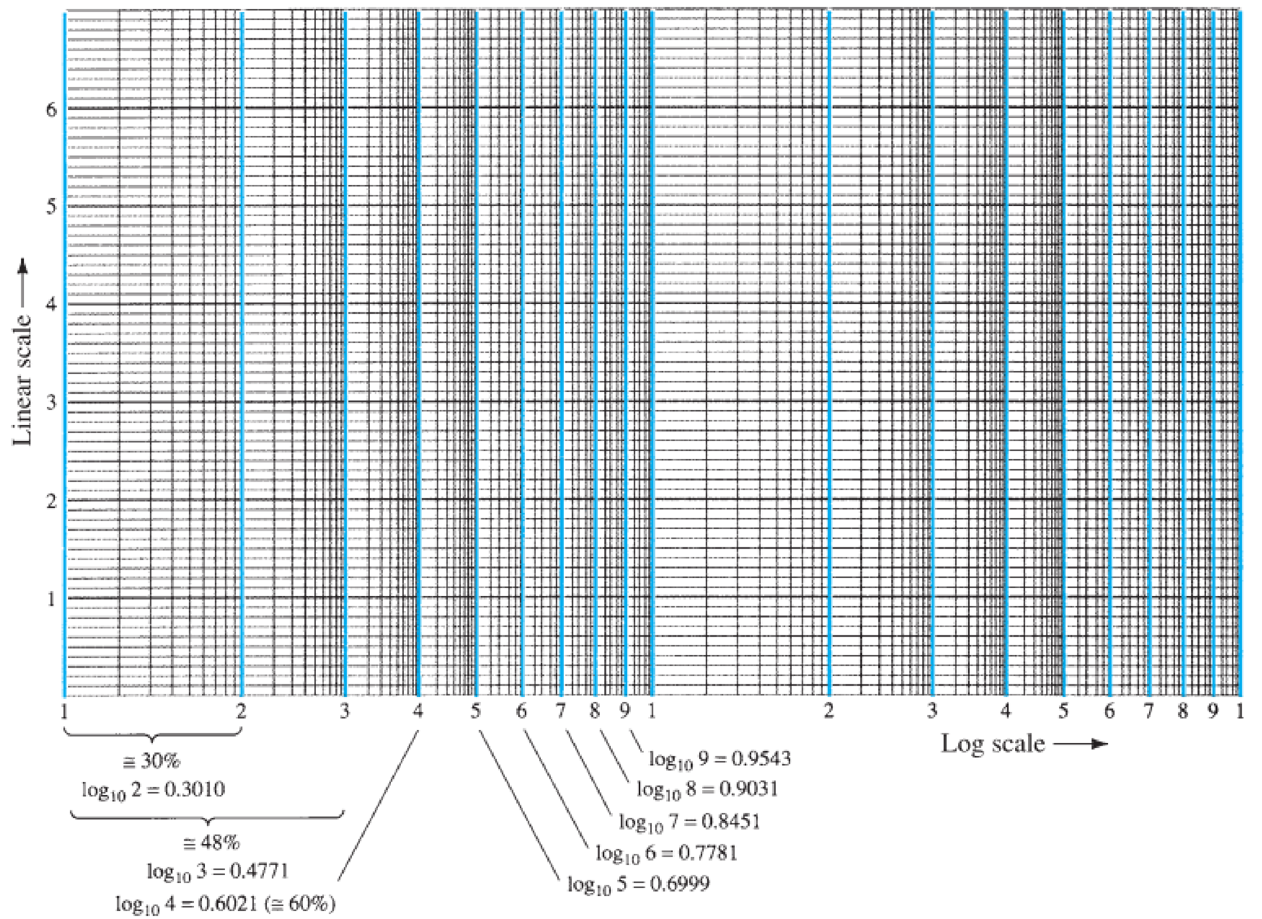
Agenda

- Introduction
- General Frequency Considerations
- Low Frequency Analysis- Bode Plot
- BJT & JFET Amplifiers Low Frequency Analysis
- Miller Effect
- BJT & JFET Amplifiers High Frequency Response

INTRODUCTION

Introduction

- We will now investigate the frequency effects introduced by the larger capacitive elements of the network at low frequencies and the smaller capacitive elements of the active device at high frequencies



Decibels

- Power Levels

$$G = \log_{10} \frac{P_2}{P_1} \quad \text{bel}$$

$$G_{dB} = 10 \log_{10} \frac{P_2}{P_1} \quad \text{dB}$$

$$G_{dBm} = 10 \log_{10} \frac{P_2}{1\text{ mW}} \bigg|_{600\,\Omega} \quad \text{dBm}$$

$$G_{dB} = 10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \frac{V_2^2/R_i}{V_1^2/R_i} = 10 \log_{10} \left(\frac{V_2}{V_1}\right)^2$$

$$G_{dB} = 20 \log_{10} \frac{V_2}{V_1} \quad \text{dB}$$

- Cascaded Stages

$$|A_{v_T}| = |A_{v_1}| \cdot |A_{v_2}| \cdot |A_{v_3}| \cdots |A_{v_n}|$$

$$G_{dB_T} = G_{dB_1} + G_{dB_2} + G_{dB_3} + \cdots + G_{dB_n} \quad \text{dB}$$

- Voltage gain versus dB levels

Comparing $A_v = \frac{V_o}{V_i}$ *to dB*

Voltage Gain, V_o/V_i	dB Level
0.5	–6
0.707	–3
1	0
2	6
10	20
40	32
100	40
1000	60
10,000	80
etc.	

GENERAL FREQUENCY CONSIDERATIONS

Low, High & Mid Frequency Range

Variation in $X_C = \frac{1}{2\pi f_C}$ with frequency for a 1-μF capacitor

f	X_C	
10 Hz	15.91 kΩ	Range of possible effect
100 Hz	1.59 kΩ	
1 kHz	159 Ω	
10 kHz	15.9 Ω	
100 kHz	1.59 Ω	Range of lesser concern (≅ short-circuit equivalence)
1 MHz	0.159 Ω	
10 MHz	15.9 mΩ	
100 MHz	1.59 mΩ	

Variation in $X_C = \frac{1}{2\pi f C}$ with frequency for a 5 pF capacitor

f	X_C	
10 Hz	3,183 MΩ	Range of lesser concern (≅ open-circuit equivalent)
100 Hz	318.3 MΩ	
1 kHz	31.83 MΩ	
10 kHz	3.183 MΩ	
100 kHz	318.3 kΩ	Range of possible effect
1 MHz	31.83 kΩ	
10 MHz	3.183 kΩ	
100 MHz	318.3 Ω	

- The larger capacitors of a system will have an important impact on the response of a system in the low-frequency range and can be ignored for the high-frequency region.
- The smaller capacitors of a system will have an important impact on the response of a system in the high-frequency range and can be ignored for the low-frequency region.
- The effect of the capacitive elements in an amplifier are ignored for the mid-frequency range when important quantities such as the gain and impedance levels are determined.

Typical Frequency Response

$$P_{o_{mid}} = \frac{|V_o^2|}{R_o} = \frac{|A_{v_{mid}} V_i|^2}{R_o}$$

$$P_{o_{HPF}} = \frac{|0.707\, A_{v_{mid}} V_i|^2}{R_o} = 0.5\, \frac{|A_{v_{mid}} V_i|^2}{R_o}$$

$$P_{o_{HPF}} = 0.5\, P_{o_{mid}}$$

$$\text{bandwidth (BW)} = f_H - f_L$$

The band frequencies define a level where the gain or quantity of interest will be 70.7% of its maximum value.

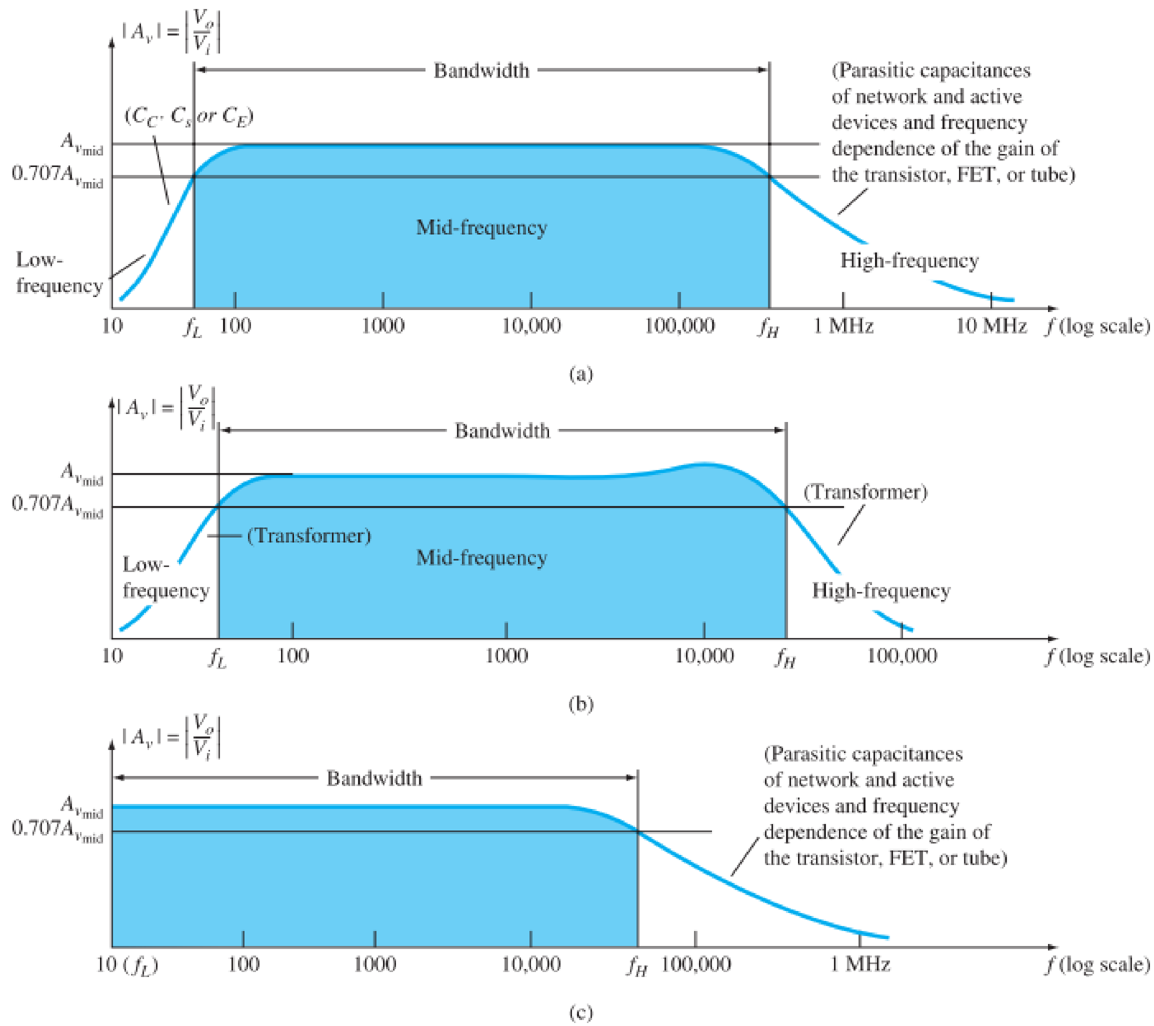


FIG. 9.8

Gain versus frequency: (a) RC-coupled amplifiers; (b) transformer-coupled amplifiers; (c) direct-coupled amplifiers.

- Normalized plot

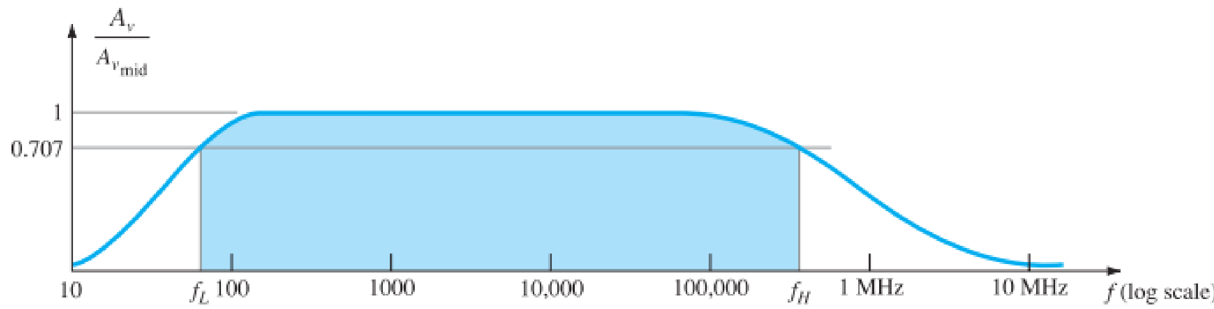


FIG. 9.9
Normalized gain versus frequency plot.

- Decibel plot

$$\left.\frac{A_v}{A_{v_{\text{mid}}}}\right|_{\text{dB}} = 20 \log_{10} \frac{A_v}{A_{v_{\text{mid}}}}$$

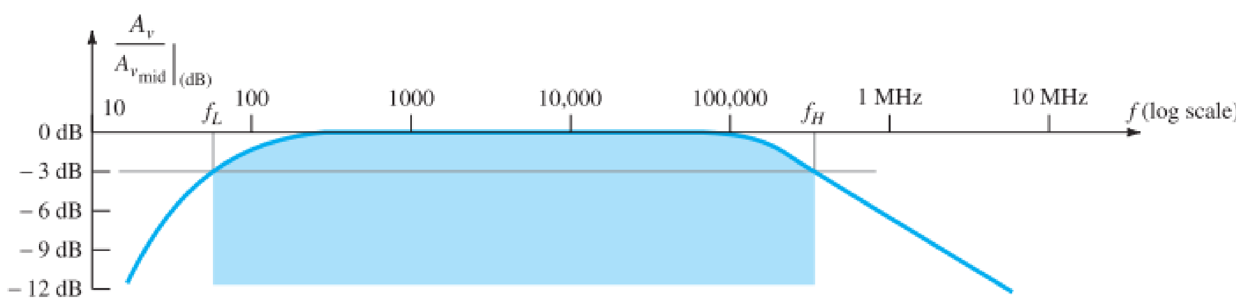


FIG. 9.12
Decibel plot of the normalized gain versus frequency plot of Fig. 9.9.

- Phase plot

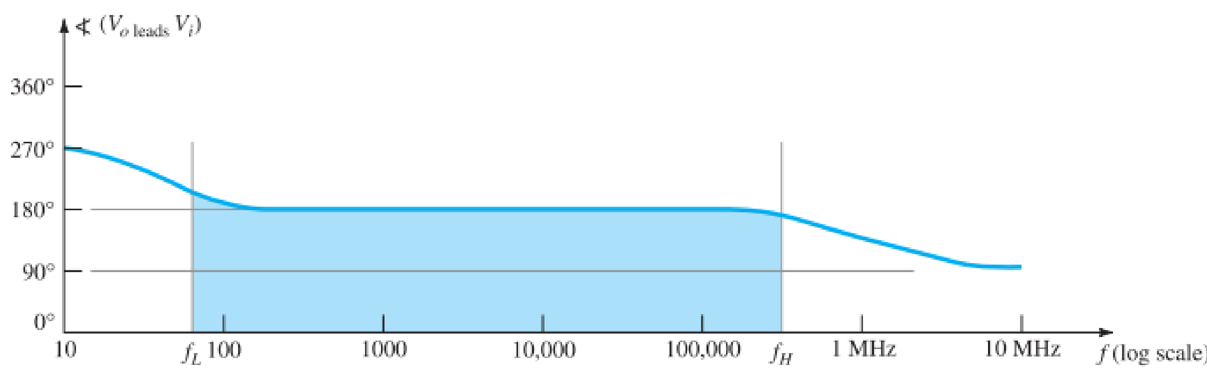


FIG. 9.13
Phase plot for an RC-coupled amplifier system.

LOW FREQUENCY ANALYSIS- BODE PLOT

Defining the Low Cutoff Frequency

- In the low-frequency region of the single-stage BJT or FET amplifier, it is the RC combinations formed by the network capacitors C_C, C_E, and C_s and the network resistive parameters that determine the cutoff frequencies

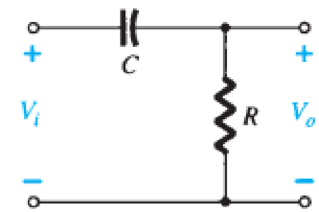


FIG. 9.14
RC combination that will define a low-cutoff frequency.

- Voltage-Divider Bias Config.

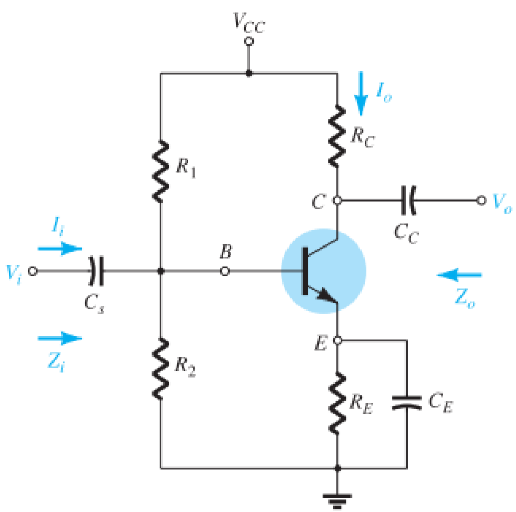


FIG. 9.15
Voltage-divider bias configuration.

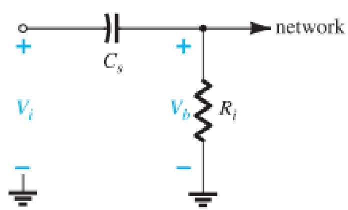


FIG. 9.16
Equivalent input circuit for the network of Fig. 9.15.

$$Z_i = R_i = R_1 \| R_2 \| \beta r_e$$

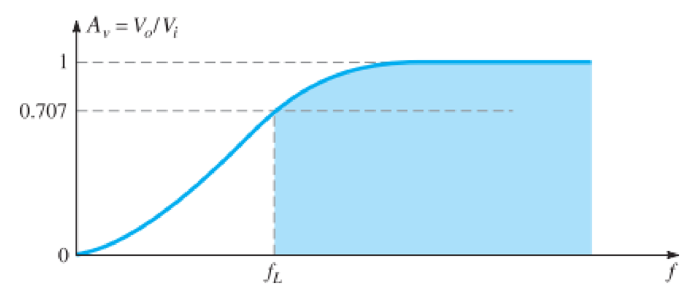


FIG. 9.19
Low-frequency response for the RC circuit of Fig. 9.14.

$$X_C = \frac{1}{2\pi f_L C} = R$$

$$f_L = \frac{1}{2\pi RC}$$

Bode Plot

$$A_{v(\text{dB})} = 20 \log_{10} \frac{1}{\sqrt{1 + (f_L/f)^2}}$$

For frequencies where $f \ll f_L$ or $(f_L/f)^2 \gg 1$,

$$A_{v(\text{dB})} = -20 \log_{10} \frac{f_L}{f}$$

$f \ll f_L$

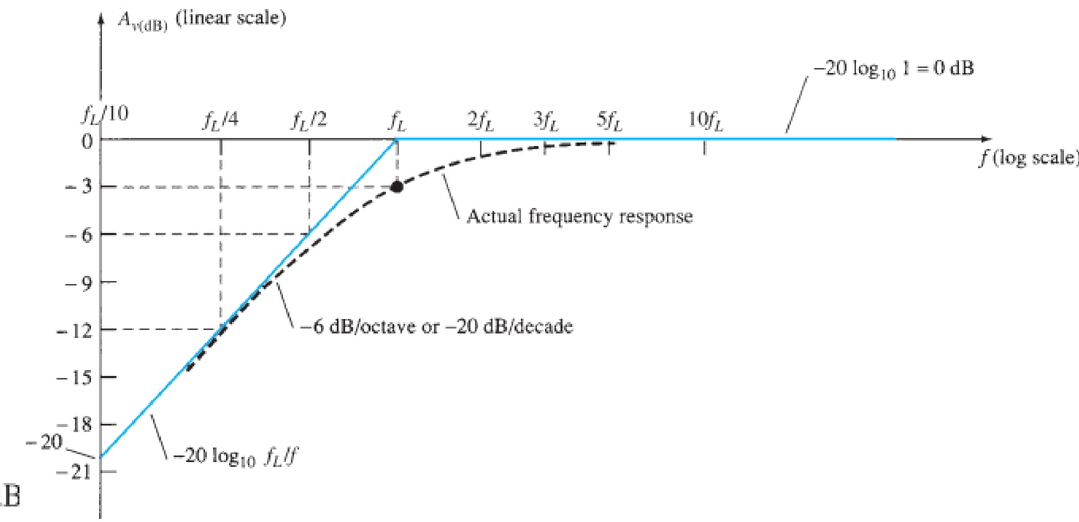
At $f = f_L$: $\frac{f_L}{f} = 1$ and $-20 \log_{10} 1 = 0$ dB

At $f = \frac{1}{2} f_L$: $\frac{f_L}{f} = 2$ and $-20 \log_{10} 2 \cong -6$ dB

At $f = \frac{1}{4} f_L$: $\frac{f_L}{f} = 4$ and $-20 \log_{10} 4 \cong -12$ dB

At $f = \frac{1}{10} f_L$: $\frac{f_L}{f} = 10$ and $-20 \log_{10} 10 = -20$ dB

- A change in frequency by a factor of two, equivalent to one octave, results in a 6-dB change in the ratio, as shown by the change in gain from $f_L/2$ to f_L.
- For a 10:1 change in frequency, equivalent to one decade, there is a 20-dB change in the ratio, as demonstrated between the frequencies of $f_L/10$ and f_L.
- The piecewise linear plot of the asymptotes and associated breakpoints is called a Bode plot of the magnitude versus frequency.



- Phase Angle:

$$\theta = \tan^{-1} \frac{f_L}{f}$$

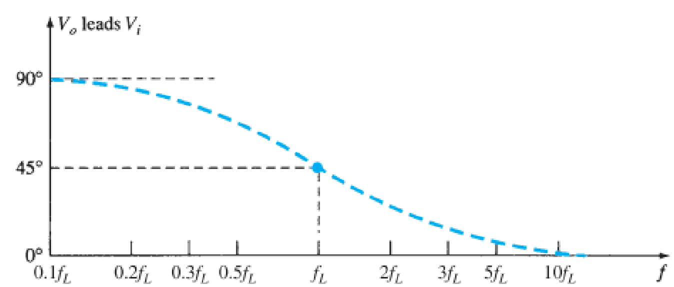


FIG. 9.22
Phase response for the RC circuit of Fig. 9.14.

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BJT & JFET AMPLIFIERS LOW FREQUENCY ANALYSIS

Loaded BJT Amplifier

In the voltage-divider ct.
→ the capacitors Cs, C_C , and C_E will determine the low-frequency response.

$f_L = \max(f_{Ls}, f_{Lc}, f_{LE})$

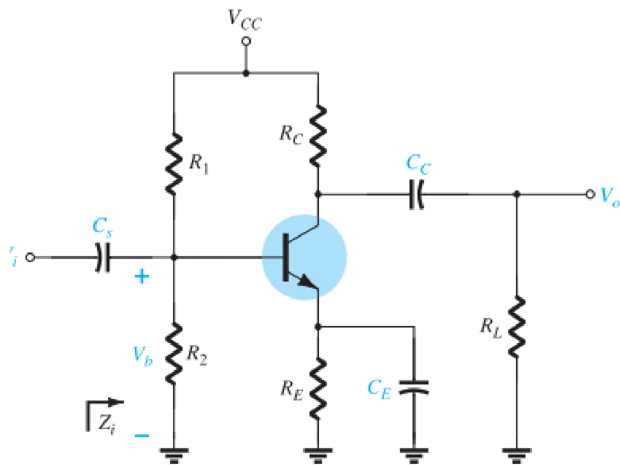


FIG. 9.25
Loaded BJT amplifier with capacitors that affect the low-frequency response.

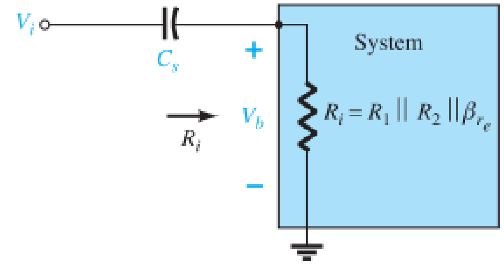


FIG. 9.26
Determining the effect of C_s on the low-frequency response.

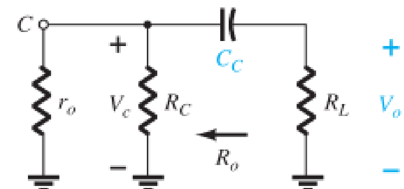


FIG. 9.28
Localized ac equivalent for C_C with $V_i = 0$ V.

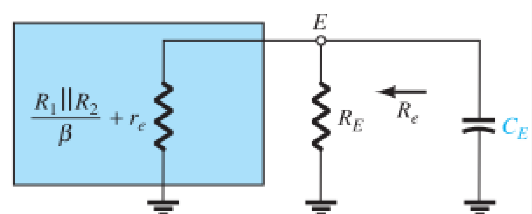


FIG. 9.30
Localized ac equivalent of C_E.

→Cs:

$$\mathbf{V}_b = \frac{R_i \mathbf{V}_i}{R_i - jX_{C_s}}$$

$$f_{L_s} = \frac{1}{2\pi R_i C_s}$$

$$R_i = R_1 \| R_2 \| \beta r_e.$$

$$\mathbf{A}_v = \frac{\mathbf{V}_b}{\mathbf{V}_i} = \frac{1}{1 - j(f_{L_s}/f)}$$

→ Cc:

$$f_{L_C} = \frac{1}{2\pi (R_o + R_L) C_C}$$

$$R_o = R_C \| r_o$$

→ C_E:

$$f_{L_E} = \frac{1}{2\pi R_e C_E}$$

$$R_e = R_E \| \left(\frac{R_1 \| R_2}{\beta} + r_e \right)$$

Impact of R_S

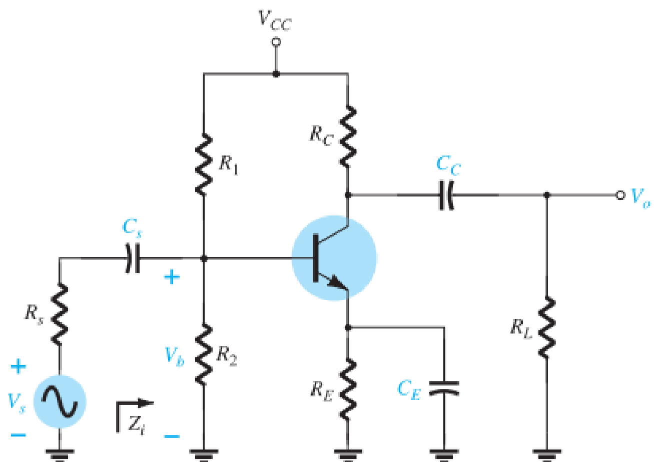


FIG. 9.32
Determining the effect of R_s on the low-frequency response of a BJT amplifier.

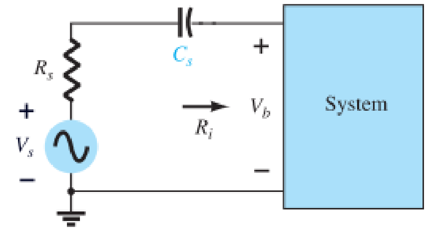


FIG. 9.33
Determining the effect of C_s on the low-frequency response.

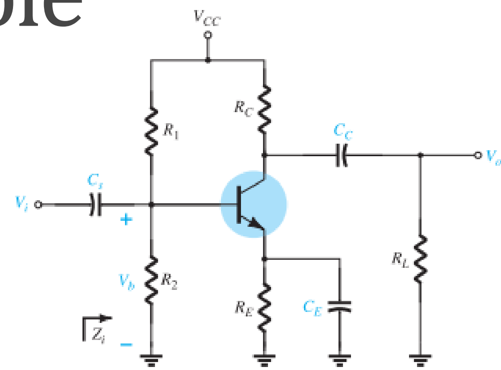
$$f_{L_s} = \frac{1}{2\pi(R_i + R_s)C_s}$$

$$f_{L_C} = \frac{1}{2\pi(R_o + R_L)C_C}$$

$$f_{L_E} = \frac{1}{2\pi R_e C_E}$$

$$R_e = R_E \| \left(\frac{R'_s}{\beta} + r_e\right) \text{ and } R'_s = R_s \| R_1 \| R_2$$

Example



$C_s = 10\ \mu F$, $C_E = 20\ \mu F$, $C_C = 1\ \mu F$
$R_1 = 40\ k\Omega$, $R_2 = 10\ k\Omega$, $R_E = 2\ k\Omega$, $R_C = 4\ k\Omega$,
$R_L = 2.2\ k\Omega$

EXAMPLE 9.12

a. Repeat the analysis of Example 9.11 but with a source resistance R_s of 1 kΩ. The gain of interest will now be V_o/V_s rather than V_o/V_i. Compare results.
b. Sketch the frequency response using a Bode plot.
c. Verify the results using PSpice.

Solution: a. The dc conditions remain the same:

$$r_e = 15.76\ \Omega \text{ and } \beta r_e = 1.576\ k\Omega$$

Midband Gain $A_v = \dfrac{V_o}{V_i} = \dfrac{-R_C \| R_L}{r_e} \cong -90$ as before

The input impedance is given by

$$Z_i = R_i = R_1 \| R_2 \| \beta r_e$$
$$= 40\ k\Omega \| 10\ k\Omega \| 1.576\ k\Omega$$
$$\cong 1.32\ k\Omega$$

and from Fig. 9.35,

$$V_b = \frac{R_i V_s}{R_i + R_s}$$

or

$$\frac{V_b}{V_s} = \frac{R_i}{R_i + R_s} = \frac{1.32\ k\Omega}{1.32\ k\Omega + 1\ k\Omega} = 0.569$$

so that

$$A_{v_s} = \frac{V_o}{V_s} = \frac{V_o}{V_i} \cdot \frac{V_b}{V_s} = (-90)(0.569)$$
$$= \mathbf{-51.21}$$

C_s

$$R_i = R_1 \| R_2 \| \beta r_e = 40\ k\Omega \| 10\ k\Omega \| 1.576\ k\Omega \cong 1.32\ k\Omega$$
$$f_{L_S} = \frac{1}{2\pi(R_s + R_i)C_s} = \frac{1}{(6.28)(1\ k\Omega + 1.32\ k\Omega)(10\ \mu F)}$$
$$f_{L_S} \cong 6.86 \text{ Hz vs. } 12.06 \text{ Hz without } R_s$$

C_C

$$f_{L_C} = \frac{1}{2\pi(R_C + R_L)C_C}$$
$$= \frac{1}{(6.28)(4\ k\Omega + 2.2\ k\Omega)(1\ \mu F)}$$
$$\cong \mathbf{25.68\ Hz} \text{ as before}$$

C_E

$$R'_s = R_s \| R_1 \| R_2 = 1\ k\Omega \| 40\ k\Omega \| 10\ k\Omega \cong 0.889\ k\Omega$$
$$R_e = R_E \left\| \left(\frac{R'_s}{\beta} + r_e\right) = 2\ k\Omega \right\| \left(\frac{0.889\ k\Omega}{100} + 15.76\ \Omega\right)$$
$$= 2\ k\Omega \| (8.89\ \Omega + 15.76\ \Omega) = 2\ k\Omega \| 24.65\ \Omega \cong 24.35\ \Omega$$
$$f_{L_E} = \frac{1}{2\pi R_e C_E} = \frac{1}{(6.28)(24.35\ \Omega)(20\ \mu F)} = \frac{10^6}{3058.36}$$
$$\cong \mathbf{327\ Hz} \text{ vs. } 87.13 \text{ Hz without } R_s.$$

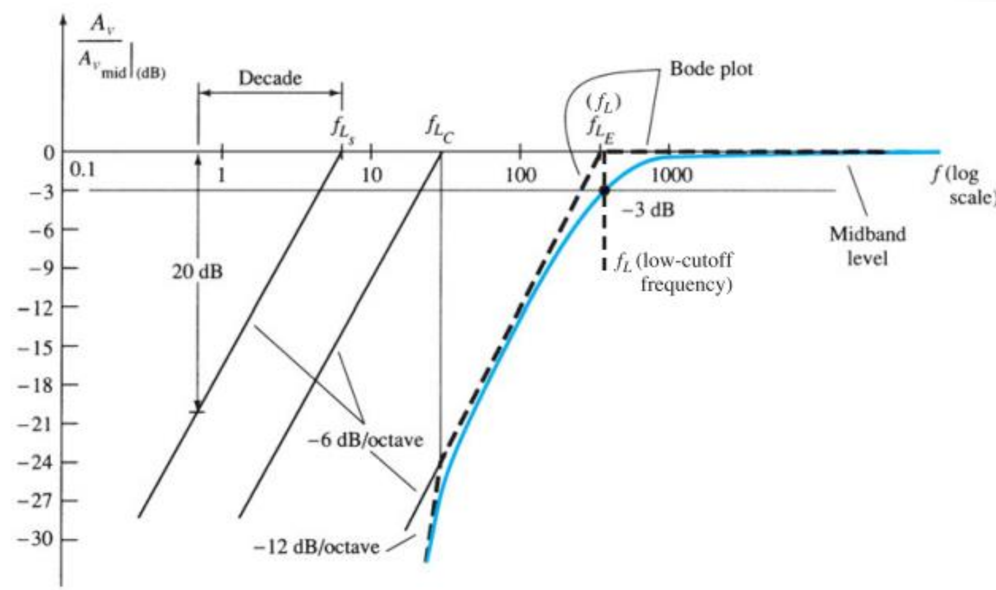
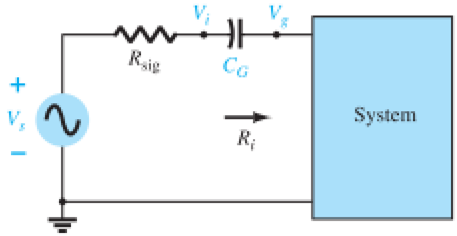


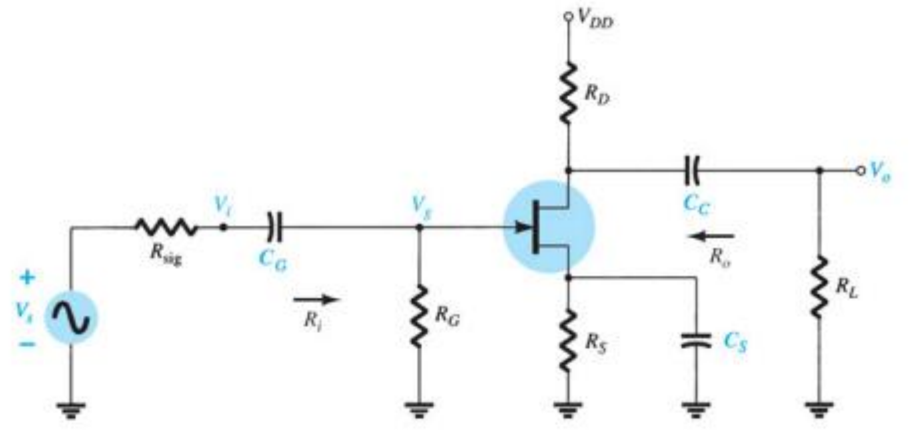
FIG. 9.36
Low-frequency plot for the network of Example 9.12.

FET Amplifier

→ $$f_{L_G} = \frac{1}{2\pi(R_{sig} + R_i)C_G}$$

$$R_i = R_G$$





→ $$f_{L_C} = \frac{1}{2\pi(R_o + R_L)C_C}$$

$$R_o = R_D \| r_d$$

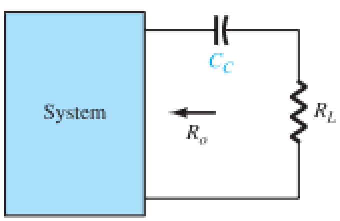


FIG. 9.39
Determining the effect of C_C on the low-frequency response.

→ $$f_{L_S} = \frac{1}{2\pi R_{eq} C_S}$$

$$R_{eq} = \frac{R_S}{1 + R_S(1 + g_m r_d)/(r_d + R_D \| R_L)}$$

$$R_{eq} = R_S \| \frac{1}{g_m}$$

$r_d \cong \infty\Omega$

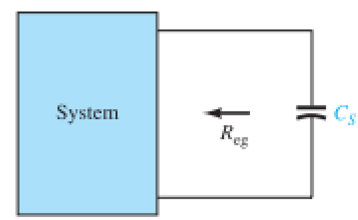


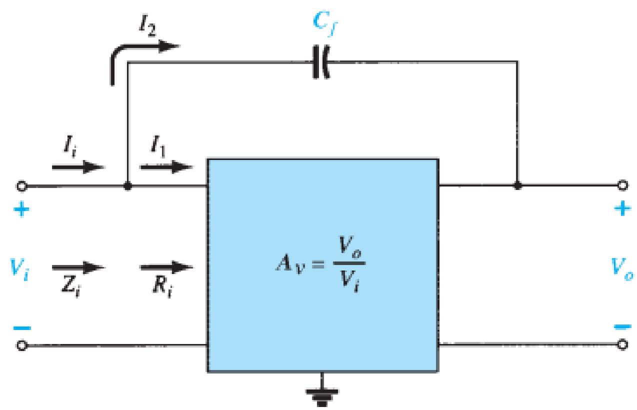
FIG. 9.40
Determining the effect of C_S on the low-frequency response.

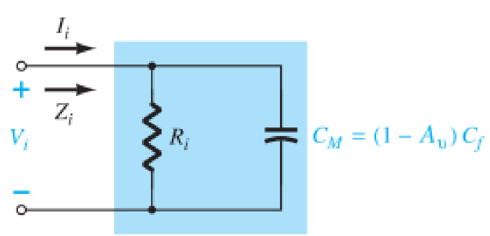
MILLER EFFECT

Miller input capacitance

$$C_{M_i} = (1 - A_v)C_f$$

- In the high-frequency region, the capacitive elements of importance are the interelectrode (between-terminals) capacitances internal to the active device and the wiring capacitance between leads of the network.
- For any inverting amplifier, the input capacitance will be increased by a Miller effect capacitance sensitive to the gain of the amplifier and the interelectrode (parasitic) capacitance between the input and output terminals of the active device.





Applying Kirchhoff's current law gives

$$I_i = I_1 + I_2$$

Using Ohm's law yields

$$I_i = \frac{V_i}{Z_i}, \quad I_1 = \frac{V_i}{R_i}$$

and

$$I_2 = \frac{V_i - V_o}{X_{C_f}} = \frac{V_i - A_v V_i}{X_{C_f}} = \frac{(1 - A_v)V_i}{X_{C_f}}$$

Substituting, we obtain

$$\frac{V_i}{Z_i} = \frac{V_i}{R_i} + \frac{(1 - A_v)V_i}{X_{C_f}}$$

and

$$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{X_{C_f}/(1 - A_v)}$$

but

$$\frac{X_{C_f}}{1 - A_v} = \frac{1}{\omega\underbrace{(1 - A_v)C_f}_{C_M}} = X_{C_M}$$

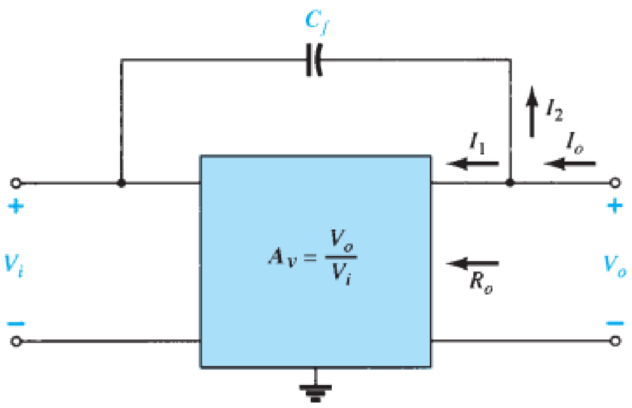
and

$$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{X_{C_M}}$$

Miller output capacitance

$$C_{M_o} = \left(1 - \frac{1}{A_v}\right)C_f$$

- A positive value for A_v would result in a negative capacitance (for Av > 1).
- For noninverting amplifiers such as the common-base and emitter-follower configurations, the Miller effect capacitance is not a contributing concern for high-frequency applications.
- The Miller effect will also increase the level of output capacitance, which must also be considered when the high-frequency cutoff is determined.



$$C_{M_o} = \left(1 - \frac{1}{A_v}\right)C_f$$

$$C_{M_o} \cong C_f \quad |A_v| \gg 1$$

$$I_o = I_1 + I_2$$

$$I_1 = \frac{V_o}{R_o} \quad \text{and} \quad I_2 = \frac{V_o - V_i}{X_{C_f}}$$

The resistance R_o is usually sufficiently large to permit ignoring the first term of the equation compared to the second term and assuming that

$$I_o \cong \frac{V_o - V_i}{X_{C_f}}$$

Substituting $V_i = V_o/A_v$ from $A_v = V_o/V_i$ results in

$$I_o = \frac{V_o - V_o/A_v}{X_{C_f}} = \frac{V_o(1 - 1/A_v)}{X_{C_f}}$$

and

$$\frac{I_o}{V_o} = \frac{1 - 1/A_v}{X_{C_f}}$$

or

$$\frac{V_o}{I_o} = \frac{X_{C_f}}{1 - 1/A_v} = \frac{1}{\omega C_f(1 - 1/A_v)} = \frac{1}{\omega C_{M_o}}$$

BJT & JFET AMPLIFIERS HIGH FREQUENCY RESPONSE

High Frequency Response

- At the high-frequency end, there are two factors that define the 3-dB cutoff point:
  1. the network capacitance (parasitic and introduced)
  2. the frequency dependence of h_{fe} (β).

- For RC circuit:

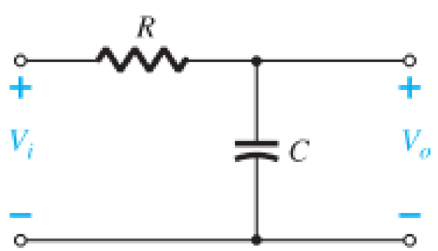
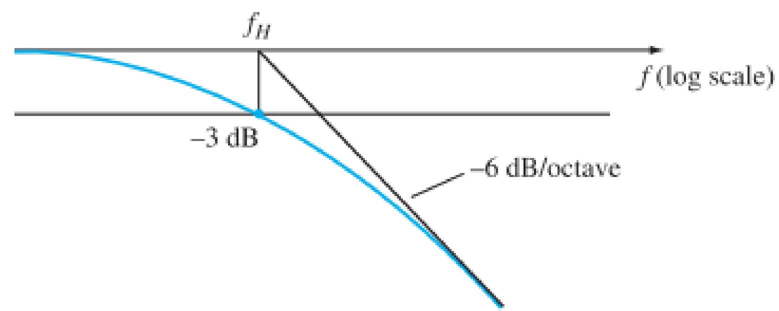


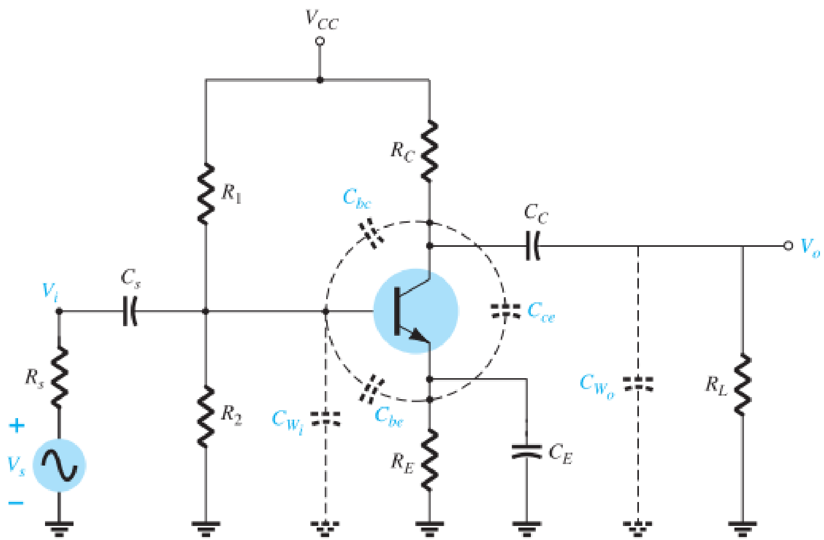
FIG. 9.45
RC combination that will define a high-cutoff frequency.



$$A_v = \frac{1}{1 + j(f/f_H)}$$

1. Network Parameters :

- At high frequencies, the various parasitic capacitances (C_{be}, C_{bc}, C_{ce}) of the transistor are included with the wiring capacitances (C_{Wi}, C_{Wo}).



$$f_{H_i} = \frac{1}{2\pi R_{Th_i} C_i}$$

$$R_{Th_i} = R_s \| R_1 \| R_2 \| \beta r_e$$

$$C_i = C_{W_i} + C_{be} + C_{M_i} = C_{W_i} + C_{be} + (1 - A_v)C_{bc}$$

$$f_{H_o} = \frac{1}{2\pi R_{Th_o} C_o}$$

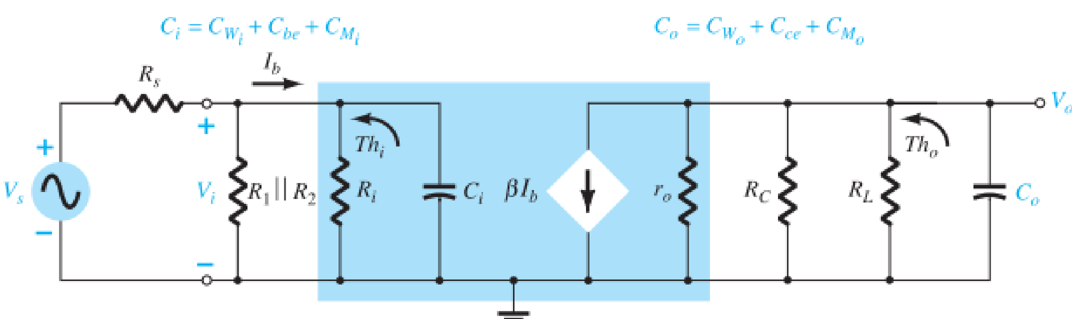
$$R_{Th_o} = R_C \| R_L \| r_o$$

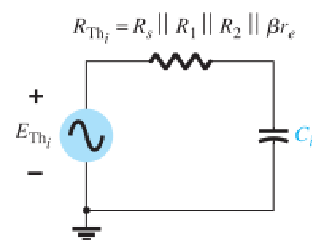
$$C_o = C_{W_o} + C_{ce} + C_{M_o}$$

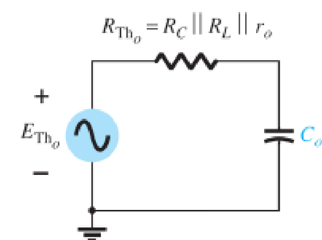
$$C_o = C_{W_o} + C_{ce} + (1 - 1/A_v)C_{bc}$$

$$1 \gg 1/A_v$$

$$C_o \cong C_{W_o} + C_{ce} + C_{bc}$$





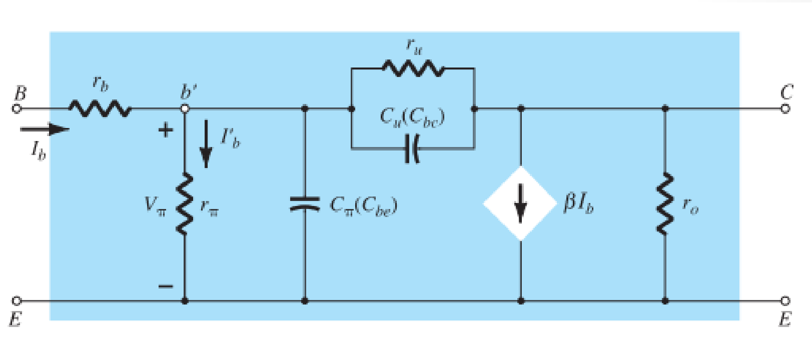


2. h_{fe} (or β) Variation

- The variation of h_{fe} (or β) with frequency approaches the following relationship:

$$h_{fe} = \frac{h_{fe_{\text{mid}}}}{1 + j(f/f_\beta)}$$

- The quantity, f_β, is determined by a set of parameters employed in the hybrid π model

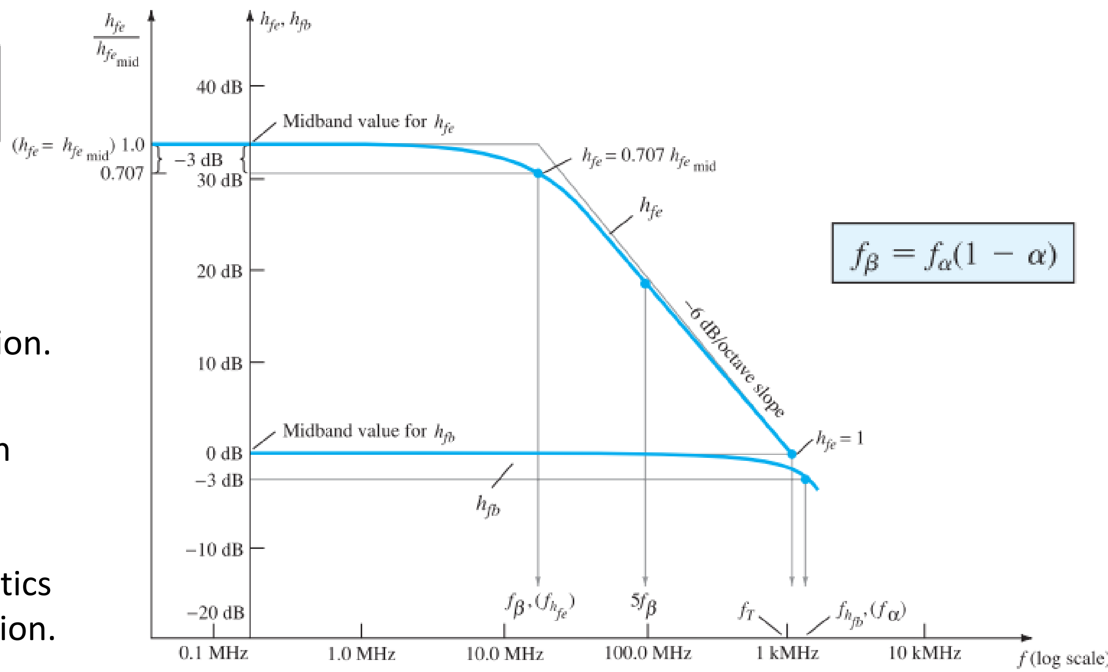


$$f_\beta(\text{often appearing as } f_{h_{fe}}) = \frac{1}{2\pi r_\pi(C_\pi + C_u)}$$

$$f_\beta = \frac{1}{h_{fe_{\text{mid}}}} \frac{1}{2\pi r_e(C_\pi + C_u)}$$

- f_β is a function of the bias configuration.

- the small change in h_{fb} for the chosen frequency range, revealing that the common-base configuration displays improved high-frequency characteristics over the common-emitter configuration.



$$f_\beta = f_\alpha(1 - \alpha)$$

FIG. 9.51
h_{fe} and h_{fb} versus frequency in the high-frequency region.

Example

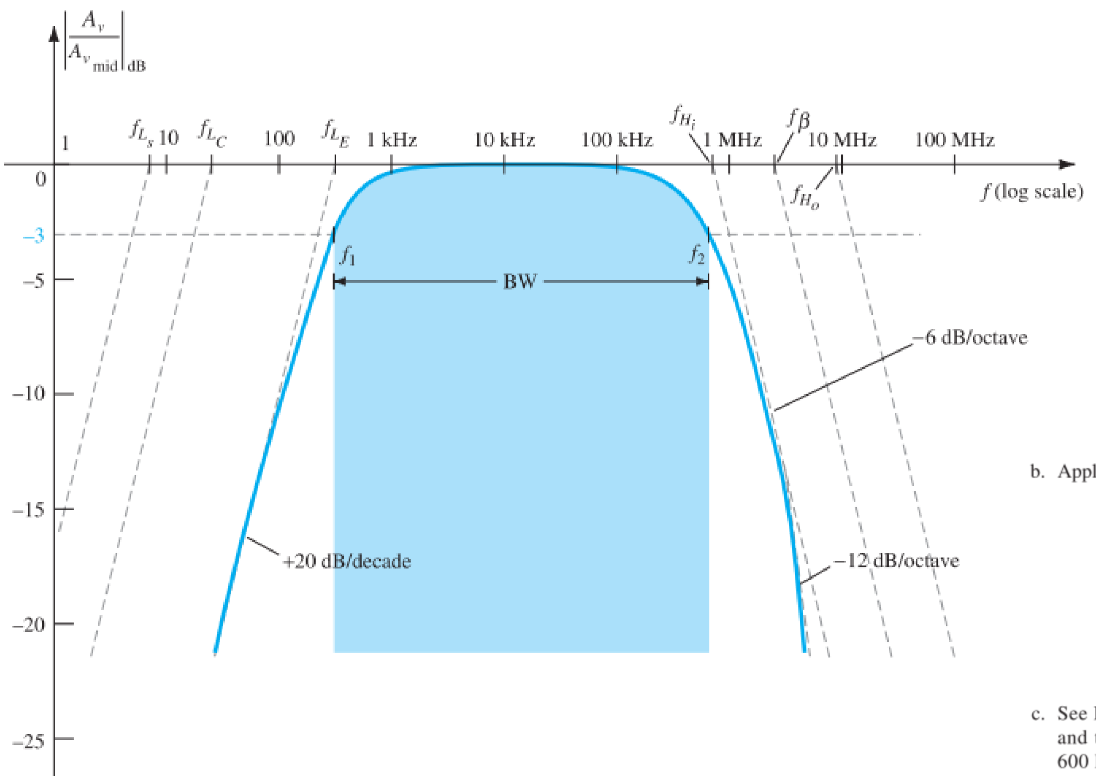
EXAMPLE 9.14 Use the network of Fig. 9.47 with the same parameters as in Example 9.12, that is,

$$R_s = 1\,\text{k}\Omega, R_1 = 40\,\text{k}\Omega, R_2 = 10\,\text{k}\Omega, R_E = 2\,\text{k}\Omega, R_C = 4\,\text{k}\Omega, R_L = 2.2\,\text{k}\Omega$$
$$C_s = 10\,\mu\text{F}, C_C = 1\,\mu\text{F}, C_E = 20\,\mu\text{F}$$
$$h_{fe} = 100, r_o = \infty\,\Omega, V_{CC} = 20\,\text{V}$$

with the addition of

$$C_\pi(C_{be}) = 36\,\text{pF}, C_u(C_{bc}) = 4\,\text{pF}, C_{ce} = 1\,\text{pF}, C_{W_i} = 6\,\text{pF}, C_{W_o} = 8\,\text{pF}$$

a. Determine f_{H_i} and f_{H_o}.
b. Find f_β and f_T.
c. Sketch the frequency response for the low- and high-frequency regions using the results of Example 9.12 and the results of parts (a) and (b).



Solution:

a. From Example 9.12:

$$\beta r_e = 1.576\,\text{k}\Omega, \quad A_{v_{mid}}(\text{amplifier—not including effects of } R_s) = -90$$

and
$$R_{Th_i} = R_s \| R_1 \| R_2 \| \beta r_e = 1\,\text{k}\Omega \| 40\,\text{k}\Omega \| 10\,\text{k}\Omega \| 1.576\,\text{k}\Omega \cong 0.57\,\text{k}\Omega$$

with
$$C_i = C_{W_i} + C_{be} + (1 - A_v)C_{bc} = 6\,\text{pF} + 36\,\text{pF} + [1 - (-90)]4\,\text{pF} = 406\,\text{pF}$$

$$f_{H_i} = \frac{1}{2\pi R_{Th_i} C_i} = \frac{1}{2\pi(0.57\,\text{k}\Omega)(406\,\text{pF})} = \mathbf{687.73\,kHz}$$

$$R_{Th_o} = R_C \| R_L = 4\,\text{k}\Omega \| 2.2\,\text{k}\Omega = 1.419\,\text{k}\Omega$$

$$C_o = C_{W_o} + C_{ce} + C_{M_o} = 8\,\text{pF} + 1\,\text{pF} + \left(1 - \frac{1}{-90}\right)4\,\text{pF} = 13.04\,\text{pF}$$

$$f_{H_o} = \frac{1}{2\pi R_{Th_o} C_o} = \frac{1}{2\pi(1.419\,\text{k}\Omega)(13.04\,\text{pF})} = \mathbf{8.6\,MHz}$$

b. Applying Eq. (9.63) gives

$$f_\beta = \frac{1}{2\pi h_{fe_{mid}} r_e (C_{be} + C_{bc})} = \frac{1}{2\pi(100)(15.76\,\Omega)(36\,\text{pF} + 4\,\text{pF})} = \frac{1}{2\pi(100)(15.76\,\Omega)(40\,\text{pF})} = \mathbf{2.52\,MHz}$$

$$f_T = h_{fe_{mid}} f_\beta = (100)(2.52\,\text{MHz}) = \mathbf{252\,MHz}$$

c. See Fig. 9.54. The corner frequency f_{H_i} will determine the high cutoff frequency and the bandwidth of the amplifier. The upper cutoff frequency is very close to 600 kHz.

Gain-Bandwidth Product

- There is a Figure of Merit applied to amplifiers called the Gain-Bandwidth Product (GBP) that is commonly used to initiate the design process of an amplifier.
- It provides important information about the relationship between the gain of the amplifier and the expected operating frequency range.

$$GBP = A_{v_{mid}} BW$$

$$BW = f_H - f_L \cong f_H$$

$$f_T = A_{v_{mid}} f_H \quad \text{(Hz)}$$

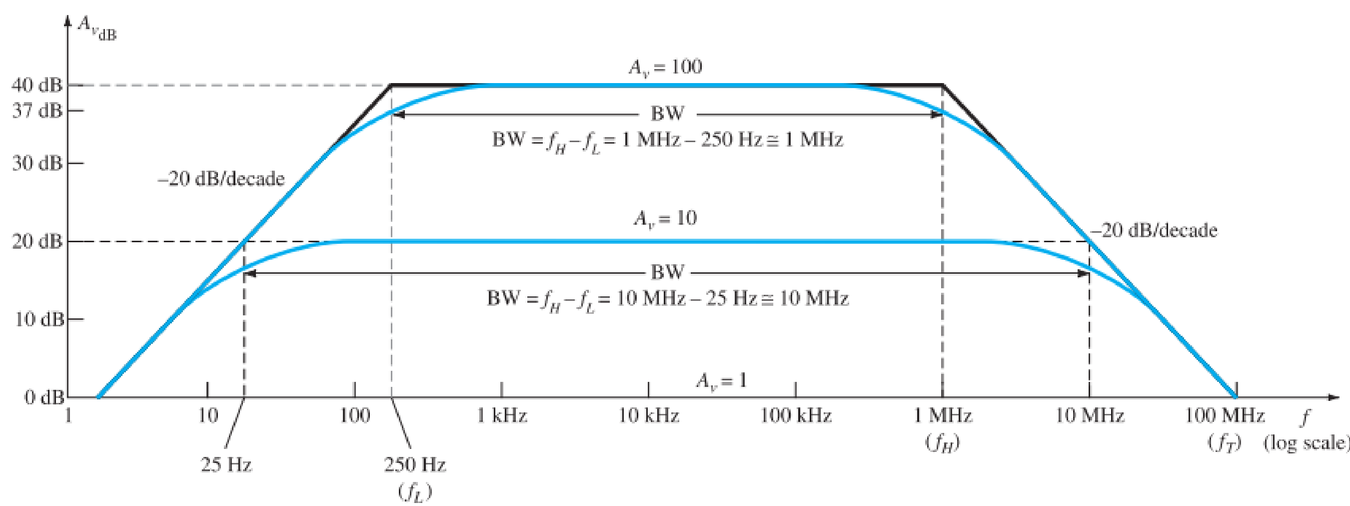
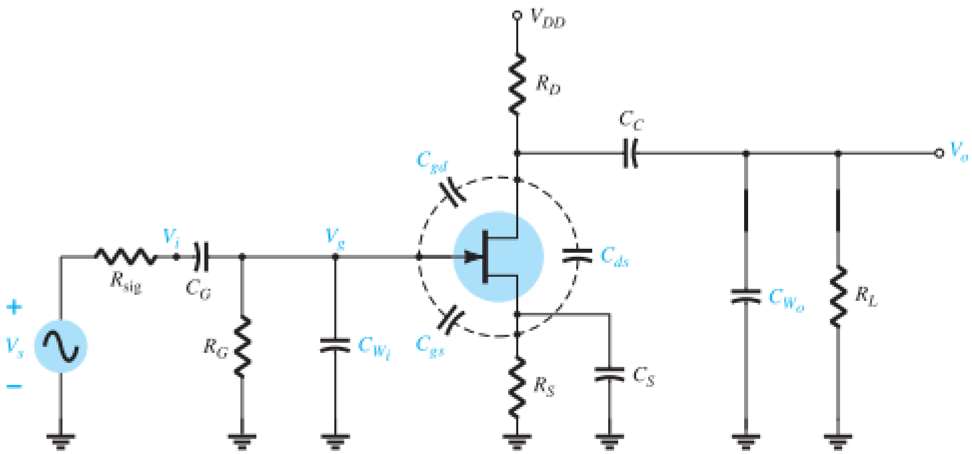


FIG. 9.53
Finding the bandwidth at two different gain levels.

- at any level of gain the product of the two remains a constant.
- the frequency f_T is called the unity-gain frequency and is always equal to the product of the midband gain of an amplifier and the bandwidth at any level of gain.

FET Amplifier

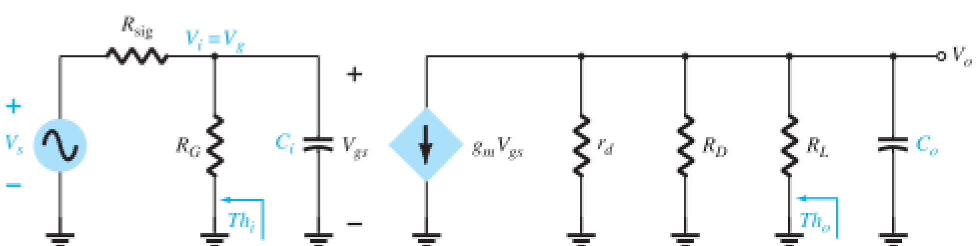


$$f_{H_i} = \frac{1}{2\pi R_{Th_i} C_i}$$

$$R_{Th_i} = R_{sig} \| R_G$$

$$C_i = C_{W_i} + C_{gs} + C_{M_i}$$

$$C_{M_i} = (1 - A_v)C_{gd}$$



$$f_{H_o} = \frac{1}{2\pi R_{Th_o} C_o}$$

$$R_{Th_o} = R_D \| R_L \| r_d$$

$$C_o = C_{W_o} + C_{ds} + C_{M_o}$$

$$C_{M_o} = \left(1 - \frac{1}{A_v}\right)C_{gd}$$

- For more details, refer to:
  - Chapter 9, Electronic Devices and Circuits, Boylestad.
- The lecture is available online at:
  - https://speakerdeck.com/ahmad_elbanna
- For inquires, send to:
  - ahmad.elbanna@feng.bu.edu.eg